

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Engineering Mathematics-II**Subject Code: 4TE02EMT2****Semester: II****Time: 10.30 To 1:30****Branch: B.Tech(All)****Date: 19/11/2015****Marks: 70**

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions:**(14)**

a) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

b) $\int_0^{\pi/2} \cos^4 x \, dx = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) $\frac{3\pi}{16}$ (d) $\frac{8\pi}{3}$

c) $\int_0^1 \int_0^x dy \, dx = \underline{\hspace{2cm}}$

- (a) $\frac{1}{2}$ (b) -1 (c) 0 (d) y

d) The value of $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$ for $m \neq \pm n$ is

- (a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

e) $\beta(1,1) = \underline{\hspace{2cm}}$

- (a) 0 (b) 1 (c) $\sqrt{\pi}$ (d) π

f) $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right) = \underline{\hspace{2cm}}$

- (a) $\frac{\sqrt{\pi} \Gamma(2n)}{2^{2n-1}}$ (b) $\frac{\sqrt{\pi} \Gamma(2n)}{2^{2n}}$ (c) $\frac{\sqrt{\pi} \Gamma(n)}{2^{2n-1}}$ (d) $\frac{\sqrt{\pi} \Gamma(n)}{2^{2n}}$



- g)** If the two tangents at the point are real and coincident, the double point is called _____.
- (a) a node (b) a cusp (c) a conjugate point (d) none of these
- h)** The curve passes through the origin, if the equation does not contain _____
- (a) terms in x (b) terms in y (c) constant term (d) none of these
- i)** Length of curve for $y = f(x)$ is defined by
- (a) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ (b) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- (c) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$ (d) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$
- j)** $\int_0^1 \int_1^2 \int_0^3 dx dy dz =$ _____
- (a) 1 (b) -3 (c) $\frac{1}{3}$ (d) 3
- k)** The degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}}$ is
- (a) 1 (b) 2 (c) 3 (d) 6
- l)** The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left[y + 5\left(\frac{dy}{dx}\right)\right]^{\frac{1}{2}}$ is
- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2
- m)** The p -series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ is convergent for
- (a) $p < 1$ (b) $p > 1$ (c) $p = 1$ (d) none of these
- n)** The series $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$ is
- (a) convergent (b) divergent (c) oscillatory (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

- a)** Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. **(05)**
- b)** Evaluate: $\int_0^{\pi} x \sin^7 x \cos^4 x dx$ **(05)**



- c) Prove that (i) $\operatorname{erfc}(x) + \operatorname{erfc}(-x) = 2$
(ii) $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$ (04)

Q-3 Attempt all questions

a) Evaluate: $\int_0^1 \left(x \log \frac{1}{x} \right)^n dx$ (05)

b) Test for the convergence the series $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$ (05)

c) Solve: $\frac{dy}{dx} + y \tan x = \sin 2x, y(0) = 1$ (04)

Q-4 Attempt all questions

a) Test for convergence the series $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \dots$ (05)

b) Trace the curve $r = a(1 + \cos \theta)$. (05)

c) Prove that (i) $n\beta(m+1, n) = m\beta(m, n+1)$
(ii) $\beta(m, n) = \beta(m, n+1) + \beta(m+1, n)$ (04)

Q-5 Attempt all questions

a) Evaluate: $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$ (05)

b) Solve: $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$ (05)

c) Test for convergence the series $4 - 1 + \frac{1}{4} - \frac{1}{16} + \dots$ and if it is convergent then also find its sum. (04)

Q-6 Attempt all questions

a) Evaluate $\iint_R xy \, dy \, dx$, where R is the positive quadrant of the circle $x^2 + y^2 = a^2$ (05)

b) Derive Reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x \, dx, n \geq 2$. (05)

c) Find the orthogonal trajectories of the family of parabola $y = ax^2$. (04)



Q-7 Attempt all questions

a) Change the order of integration and evaluate $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx dy$. (05)

b) A circuit containing a resistance R , an inductance L in series is acted on by periodic electromotive force $E \sin \omega t$. If $i = 0$ when $t = 0$, show that the current at any time t is $i(t) = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \left\{ \sin(\omega t - \phi) + e^{-\frac{Rt}{L}} \sin \phi \right\}$, where $\phi = \tan^{-1} \left(\frac{L\omega}{R} \right)$ (05)

c) Solve: $(y^2 e^{xy^2} + 4x^3) dx + (2xy e^{xy^2} - 3y^2) dy = 0$ (04)

Q-8 Attempt all questions

a) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{2ax - x^2} \sqrt{a^2 - x^2}}$ (05)

b) Trace the curve $y^2(2a - x) = x^3$. (05)

c) Find the perimeter of the cardioid $r = a(1 + \cos \theta)$ (04)

